Enrollment No: _____ Exam Seat No:_____

C.U.SHAH UNIVERSITY

Summer Examination-2016

Subject Name : Engineering Mathematics-I
Subject Code : 4TE01EMT2 Branch : B.Tech (All)

Semester : 1 Date :21/04/2016 Time :10:30 To 1:30 Marks : 70

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) The polar form of complex number $\frac{1+i}{1-i}$.

a) $\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}$ b) $\sin\frac{\pi}{2}+i\cos\frac{\pi}{2}$ c) $\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}$ d) $\sin\frac{\pi}{4}+i\cos\frac{\pi}{4}$

b) The Imaginary part of Complex number e^{3z} is

a) $e^{y}\sin x$ b) $e^{x}\cos y$ c) $e^{3x}\cos 3y$ d) $e^{3x}\sin 3y$

c) $\lim_{x\to 0}\frac{\tan x}{x}=$ _____.

a) 0 b) 1 c) ∞ d)-1

d) $\lim_{x\to 0}\frac{x-\sin x}{x}=$ _____.

a) 0 b) 1 c) ∞ d)-1

e) The rank of diagonal matrix $\begin{bmatrix}1 & 0 & 0\\ 0 & -2 & 0\\ 0 & 0 & 3\end{bmatrix}$ is

a) 0 b) 1 C) 3 d) -2

f) If the rank of the matrix $\begin{bmatrix}1 & 0 & 0\\ 0 & -2 & 0\\ 0 & 0 & 3\end{bmatrix}$ is 3.then x is not equal to

a) 3 b) 4 c) 5 d) 6

g) If the power of x & y both are even ,then the curve is symmetrical about

b) X-axis b) Y-axis c)about both X & Y axes d)None of these

h) If the two tangents at the point are real & coincide, the double point is called

a) a node b) a cusp c) a conjugate point d)None of these

i) The series $1-\frac{x^2}{2!}+\frac{x^4}{4!}-.....$ represents expansion of

a) $\sin x$ b) $\cos x$ c) $\cosh x$ d) $\sinh x$

j) If $y = \cos^{-1} x$, then x=……

a) $1-\frac{y^2}{2!}+\frac{y^4}{4!}-....$ b) $1+\frac{y^2}{2!}+\frac{y^4}{4!}+....$ c) $y-\frac{y^3}{3!}+\frac{y^5}{5!}-....$ d) None of these

k) If $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=.....$

a) u b) 2u c) tan u d) sin u

l) If $p = r\tan\theta$ then $\frac{\partial p}{\partial r}$ is

a) $\tan\theta + r\sec^2\theta$ b) $\sec^2\theta$ c) $\tan\theta$ d) None of these

m) If $x = r\cos\theta, y = r\sin\theta, z = z$ *then* $\frac{\partial(x,y,z)}{\partial(r,\theta,z)}=....$

a) $\frac{1}{r}$ b) $r^2\sin\theta$ c) r d) $r^2\cos\theta$

n) If $\frac{\partial(u,v)}{\partial(x,y)} * \frac{\partial(x,y)}{\partial(u,v)}=....$

a) 0 b) -1 c) 1 d) None of these

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

A i) Find modulus and principal argument of $z = \frac{1-7i}{(3+4i)}$. **(03)**

ii) Simplify $\frac{(\cos 3\theta + i\sin 3\theta)^8 (\cos\theta - i\sin\theta)^5}{(\cos 2\theta + i\sin 2\theta)^{-2} (\cos 5\theta - i\sin 5\theta)^{-3}}$ **(04)**

B (i) Find and plot the fourth root of unity on the circle. **(07)**

Q-3 **Attempt all questions** **(14)**

A i) Evaluate: $\lim_{x\to 0}\left[\frac{1^x+2^x+3^x}{3}\right]^{\frac{1}{x}}$. **(04)**

ii) Find Maclaurin's Series of $f(x) = \sin x$. **(03)**

B i) Show that $f(x,y) = \begin{cases} \frac{xy}{x^2+y^2}; & (x,y)\neq(0,0) \\ 0 \quad ; & (x,y)=(0,0) \end{cases}$ is continuous at every point except at the origin. **(04)**

ii) Evaluate: $\lim_{x\to 0}(\cos x)^{\cot x}$.. **(03)**

Q-4 **Attempt all questions** **(14)**

A Trace the curve **(Cissoid of Diocle)** $y^2(2a-x) = x^3$. **(07)**

B If $y=\frac{x}{x^2+a^2}$. *find* y_n. **(07)**

Q-5 **Attempt all questions** **(14)**

A Find the Eigen Values & Eigen Vectors of the Matrix $A=\begin{bmatrix}11 & -4 & -7\\ 7 & -2 & -5\\ 10 & -4 & -6\end{bmatrix}$ **(07)**

B State Caley Hamilton Theorem .Verify Caley Hamilton Theorem for the matrix $A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$ & Hence, find A^{-1} **(07)**

Q-6 **Attempt all questions** **(14)**

A (i) Find the rank of matrix $A=\begin{bmatrix}2 & 3 & 4 & 5\\ 3 & 4 & 5 & 6\\ 4 & 5 & 6 & 7\\ 9 & 10 & 11 & 12\end{bmatrix}$ **(03)**

(ii) Find the inverse of the matrix $A=\begin{bmatrix}-1 & 1 & 2\\ 3 & -1 & 1\\ -1 & 3 & 4\end{bmatrix}$ using Guass Jordan Method **(04)**

B $x+y+z=6$

(i) Check the consistency of the system $x+2y+3z=14$. **(04)**

$x+4y+9z=36$

(ii) Check the following set of vectors (1,0,1),(1,1,0),(1,-1,1,(1,2,-3) is Linearly Independent or dependent ? **(03)**

Q-7 **Attempt all questions** **(14)**

A (i) If $u=\log\left(x^3+y^3-x^2y-xy^2\right)$. then Prove that $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}\right)^2 u=-\frac{4}{(x+y)^2}$. **(05)**

(ii) Find the values of $\frac{\partial f}{\partial x}$ & $\frac{\partial f}{\partial y}$ at the point (4,-5).if $f(x,y)=x^2+3xy+y-1$. **(02)**

B (i) If $u=\sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$. Prove that **(07)**

a) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}\tan u$

b) $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{1}{4}\left(\tan^3 u-\tan u\right)$.

Q-8 **Attempt all questions** **(14)**

A (i) Find Maxima & Minima of the function $x^3+y^3-3x-12y+20$. **(05)**

(ii) If $x = r\cos\theta, y = \sin\theta$ then find $\frac{\partial(x,y)}{\partial(r,\theta)}$. **(02)**

B

(i) Find the Taylor's series expansion of $f(x) = \tan^{-1}\left(\frac{y}{x}\right)$ in powers of (x-1) & (y-1). **(05)**

(ii) Find the equations of tangent plane & normal line at the point (-2,2,-3) to the ellipsoid $\frac{x^2}{4} + y^2 + \frac{z^2}{9} - 3 = 0$ **(02)**